Applying Set Partitioning Methods in the Construction of Operating Theatre Schedules

Rhyd Lewis

with Elizabeth Rowse*, Paul Harper, and Jonathan Thompson

School of Mathematics, Cardiff University, Wales.

The problem

- Across the UK, many hospital operations are being cancelled
- A significant proportion are cancelled due to lack of post-op beds on hospital wards
- In 2012/13, 18% of operations were cancelled after being scheduled the University Hospital Wales
- Of these cancellations, over 54% were due to lack of beds

*"The RCS report said **more than 2,000 non-emergency operations had not been scheduled or cancelled because of a lack of beds** in the first three months of 2013 and appears to be an inability to admit patients for elective surgery."*

South Wales Echo

Considerations

- Operating theatres (OT) are very valuable resources and should be used efficiently
- Not all theatres are suitable for all specialties
- Clinical specialties are assigned to operating theatres in a (usually weekly) cyclic schedule.
- Post-op lengths of stay can vary widely from person to person and depending on the type of operation
- OTs are a driver for demand in many other hospital departments

MAIN THEATRE

	Theatre 0 x 6759	Theatre 1 x 3071	Theatre 2 x 3072	Theatre 3 x 3073	Theatre 4 x 3074	Theatre 5 x 3075
MONDAY AM Consultant/SIT	Trauma SERVICE	Scoliosis GAW	ENT Mr G Williams SERVICE	Renal Surgery Mr Asderakis CD	Oral Surgery Mr Cronin SJP	CEPOD SR
PM Consultant/SIT	Trauma SL	Scoliosis GAW	ENT Mr S Quine SERVICE	Renal Surgery Mr Asderakis CD	Oral Surgery Mr Cronin SJP	CEPOD SR
	4pm - 9pm	Colorectal to use either TH 1 or TH 3				
TUESDAY AM Consultant/SIT	Trauma TD	Vascular Miss Hill SR	Ophthalmology Mr Watts (Paeds)	Vascular Mr I Williams SL	Paeds General Mr Surana	CEPOD LR
PM Consultant/SIT	Trauma TD	Vascular Miss Hill SR	Ophthalmology Mr Watts (Paeds)	Vascular Mr I Williams SL	Paeds General Mr Surana	CEPOD LR
	4pm - 9pm					
WEDNESDAY AM Consultant/SIT	Trauma LW	Paeds Ortho Mr Thomas	ENT Mr A Tomkinson MNC	General Surg Mr G Clarke PSS	Paeds Surgery Mr Hutton	CEPOD KM

Stages in OT scheduling

1. Case mix planning — Hospital managers
   - Divide available OT time between different surgical specialties
   - Estimate future demands for specialties
2. Master surgery schedule — Theatre department manager
   - Specify blocks of time for each specialty
     - Number of OTs and hours available for each specialty
   - Cyclic timetable is created and used on a weekly basis
3. Elective patient scheduling — Secretaries
   - Sequencing of individual surgeries in each OT
   - Based on surgeon preference, maximise utilisation etc.
   - Operational decisions

Master Surgery Schedule

Blocks of time are allocated to each surgical specialty

- MSS is a cyclic weekly timetable of surgical procedure types
- Common objectives
  - Maximise volume of patients/throughput
  - Minimise difference in target and realised OT utilisation
  - Minimise cost
- Common constraints
  - Daily availability of staff and surgical equipment
  - Ensure each specialty is assigned a minimum number of OTs for their surgeons

MAIN THEATRE

	Theatre 0 x 6759	Theatre 1 x 3071	Theatre 2 x 3072	Theatre 3 x 3073	Theatre 4 x 3074	Theatre 5 x 3075
MONDAY AM Consultant/SIT	Trauma SERVICE	Scoliosis GAW	ENT Mr G Williams SERVICE	Renal Surgery Mr Asderakis CD	Oral Surgery Mr Cronin SJP	CEPOD SR
PM Consultant/SIT	Trauma SL	Scoliosis GAW	ENT Mr S Quine SERVICE	Renal Surgery Mr Asderakis CD	Oral Surgery Mr Cronin SJP	CEPOD SR
	4pm - 9pm	Colorectal to use either TH 1 or TH 3				
TUESDAY AM Consultant/SIT	Trauma TD	Vascular Miss Hill SR	Ophthalmology Mr Watts (Paeds)	Vascular Mr I Williams SL	Paeds General Mr Surana	CEPOD LR
PM Consultant/SIT	Trauma TD	Vascular Miss Hill SR	Ophthalmology Mr Watts (Paeds)	Vascular Mr I Williams SL	Paeds General Mr Surana	CEPOD LR
	4pm - 9pm					
WEDNESDAY AM Consultant/SIT	Trauma LW	Paeds Ortho Mr Thomas	ENT Mr A Tomkinson MNC	General Surg Mr G Clarke PSS	Paeds Surgery Mr Hutton	CEPOD KM

Scheduling model

1. Find optimal schedule that satisfies
   - OT constraints – one specialty in each OT during each session
   - Bed constraints – each day, the number of empty beds on the wards determines the number of patients that can be operated on. Certain wards are only available for certain specialties.

2. Simulate schedule
   - Test new schedule with realistic predictions of lengths of stay
   - How good is the schedule?
   - Give confidence to managers/staff/patients

Optimisation model

1. List of scheduling rules
2. Generate all possible schedules for each specialty
3. Predict bed occupancy for each possible schedule
4. Select a subset of schedules (one for each specialty) that satisfy all OT and bed constraints

$$\begin{array}{lll} \min & z = c^T x & \rightarrow \text{Minimise 'cost' of chosen schedule} \\ \text{s.t.} & Ax = e & \rightarrow \text{Only one specialty scheduled in each OT session} \\ & Bx \leq d & \rightarrow \text{Don't exceed number of beds on wards} \\ & x \in \{0,1\}^n & \rightarrow \text{Select schedule or not} \end{array}$$

A Matrix

$$
\begin{aligned}
\min \quad & z = c^T x \\
\text{s.t.} \quad & Ax = e \\
& Bx \leq d \\
& x \in \{0,1\}^n
\end{aligned}
$$

***n* possible plans**

***s* specialties**

***m* OR sessions**

			Specialty 1			**Specialty 2**				**Specialty 3**			**...**				
			1	1	1	0	0	0	0	0	0	0				=	1
			0	0	0	1	1	1	1	0	0	0				=	1
			0	0	0	0	0	0	0	1	1	1				=	1
			0	0	0	0	0	0	0	0	0	0				=	1
OR 1	Mon	AM	1	1	0	1	0	1	0							≤	1
		PM	1	0	0	0	0	1	0							≤	1
OR 1	Tues	AM	0	1	0	1	1	1	0							≤	1
		PM	0	0	0	0	1	1	1							≤	1
OR 1	Weds	AM	0	0	0	0	0	0	1							≤	1
...	...	...															
OR 14	Fri	AM	0	0	1	0	0	0	1							≤	1
		PM	0	0	1	0	0	0	1							≤	1

B Matrix

$$
\begin{aligned}
\min \quad & z = c^T x \\
\text{s.t.} \quad & Ax = e \\
& Bx \le d \\
& x \in \{0,1\}^n
\end{aligned}
$$

		***n* possible plans**														
		Specialty 1			**Specialty 2**				**Specialty 3**			**…**				
Ward 1	Mon	9	9	8	0	0	0	0	0	0	0				≤	d_1
	Tue	8	7	6	0	0	0	0	0	0	0				≤	d_1
	…														…	
	Sun	4	3	3	0	0	0	0	0	0	0				≤	d_1
Ward 2	Mon	0	0	0	5	5	6	5	0	0	0				≤	d_2
	Tue	0	0	0	5	4	5	4	0	0	0				≤	d_2
	…														…	
	Sun	0	0	0	4	3	3	4	0	0	0				≤	d_2
Ward *s*																

Optimisation model

$$\begin{aligned} \min \quad & z = c^T x \\ \text{s.t.} \quad & Ax = e \\ & Bx \leq d \\ & x \in \{0,1\}^n \end{aligned}$$

$$\min z = \sum_{j=1}^{n} c_j x_j \qquad \text{(i)}$$

$s.t.$

$$\sum_{j=1}^{n} a_{ij} x_j = 1 \qquad \forall \quad i = 1, ..., s \qquad \text{(ii)}$$

$$\sum_{j=1}^{n} a_{ij} x_j \leq 1 \qquad \forall \quad i = s+1, ..., m \qquad \text{(iii)}$$

$$\sum_{j=1}^{n} b_{kj}^{(l)} x_j - \sum_{v=1}^{p} w_{kv} s_{vk}^{(l)} + \sum_{v=1}^{p} w_{vk} s_{kv}^{(l)} = d_k^{(l)} \qquad \forall \quad k = 1, ..., p, \quad l = 1, ..., q \qquad \text{(iv)}$$

$$\sum_{k=1}^{p} \sum_{v=1}^{p} w_{kv} s_{vk}^{(l)} \leq \sum_{k=1}^{p} \sum_{v=1}^{p} w_{vk} s_{kv}^{(l)} \qquad \forall \quad l = 1, ..., q \qquad \text{(v)}$$

$$\alpha_k d_k^{(l)} \leq \sum_{v=1}^{p} w_{vk} s_{kv}^{(l)} \leq \beta_k d_k^{(l)} \qquad \forall \quad k = 1, ..., p, \quad l = 1, ..., q \qquad \text{(vi)}$$

$$\alpha_k \leq \beta_k \text{ and } \alpha_k, \beta_k \in [0, 1] \qquad \forall \quad k = 1, ..., p \qquad \text{(vii)}$$

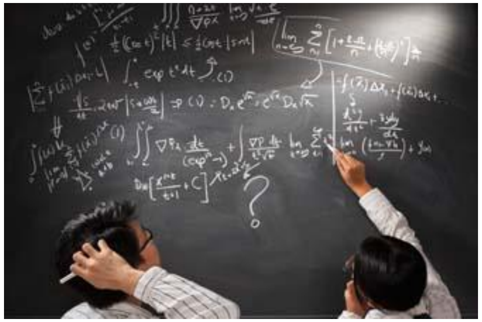
$$x_j \in \{0, 1\} \qquad \forall \quad j = 1, ..., n \qquad \text{(viii)}$$

$$s_{kv}^{(l)} \geq 0 \text{ and integer} \qquad \forall \quad k = 1, ..., p, \quad v = 1, ..., p, \quad l = 1, ..., q \qquad \text{(ix)}$$

Robust optimisation

What does the **hospital** consider a robust schedule?

- “A schedule that will accommodate fluctuations in demand for OT time and that will not cause peaks in demand for beds on wards”



A mathematically robust schedule:

- Guard against the uncertain bed count within the mathematical model
- Possibly willing to accept a compromise schedule in order to ensure the solution remains feasible when the data changes

Robust scenario approach

- Create *t* instances of the B matrix (scenarios)

- The more of these matrices that an optimal schedule can satisfy, the more **robust** it is

- For *t* scenarios, *p* wards and *q* days there are:
    - *t* x *p* x *q* bed constraints

- *t* is user defined.
    - However, there is a trade-off between a more robust schedule and having too many constraints/an over constrained problem

NHS in Cardiff

- Cardiff and Vale University Health Board
  - Teaching health board with strong links with Cardiff University

- Serves ~445,000 people in Cardiff and the Vale of Glamorgan in Wales

- University Hospital of Wales (UHW)
  - Largest hospital in Wales
  - 14 operating rooms
  - 18 surgical specialties

- ~20,000 inpatient operations/year

Single scenario results

- Optimal schedule found from single scenario based optimisation

- Current MSS:

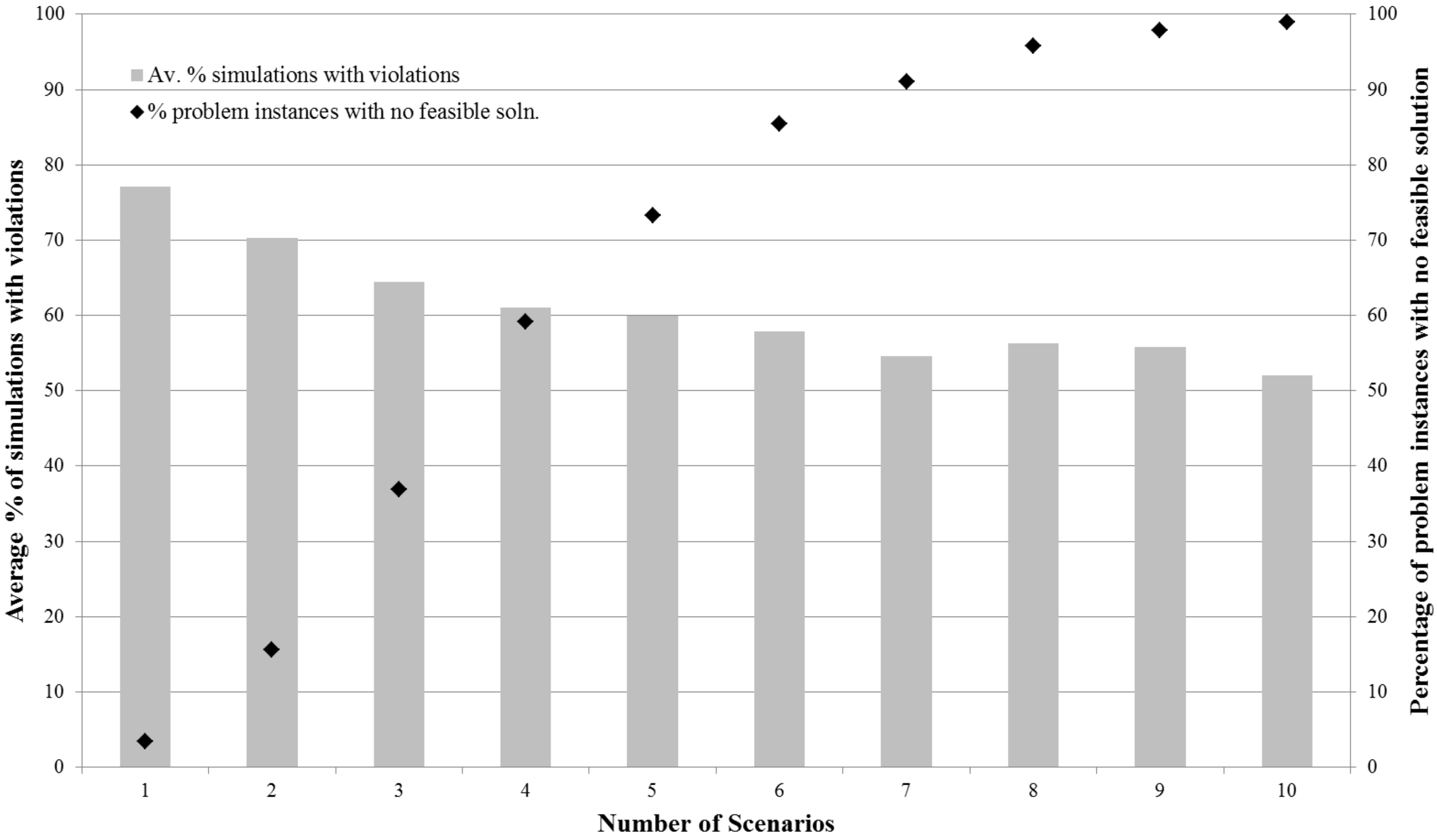
	Theatre 0	Theatre 1	Theatre 2	Theatre 3	Theatre 4	Theatre 5	Theatre 6	Theatre 7	Theatre 8	Theatre 9	Theatre 10	Theatre 11	Theatre 12	Theatre 14
MonAM	Trauma	Scoliosis	ENT	Renal	Oral	CEPOD	Urology	Colorectal	General	Thoracic	Cardiac	Cardiac	Neuro	Neuro
MonPM	Trauma	Scoliosis	ENT	Renal	Oral	CEPOD	Urology	Colorectal	General	Thoracic	Cardiac	Cardiac	Neuro	Neuro
TuesAM	Trauma	Vascular	Ophthal	Vascular	Paeds Gen	CEPOD	Urology	Colorectal	Renal	Thoracic	Cardiac	Cardiac	Neuro	Neuro
TuesPM	Trauma	Vascular	Ophthal	Vascular	Paeds Gen	CEPOD	Urology	Colorectal	Renal	Thoracic	Cardiac	Cardiac	Neuro	Neuro
WedAM	Trauma	Paeds Ortho	ENT	General	Paeds Gen	CEPOD	Urology	Colorectal	General	Thoracic	Cardiac	Cardiac	Neuro	Neuro
WedPM	Trauma	Paeds Ortho	ENT	General	Paeds Gen	CEPOD	Urology	Colorectal	General	Thoracic	Cardiac	Cardiac	Neuro	Neuro
ThurAM	Trauma	Trauma	Oral	Vascular	Paeds Gen	CEPOD	Urology	Colorectal	General	Thoracic	Cardiac	Cardiac	Neuro	Neuro
ThurPM	Trauma	Trauma	Oral	Vascular	Paeds Gen	CEPOD	Urology	Colorectal	General	Thoracic	Cardiac	Cardiac	Neuro	Neuro
FriAM	Trauma	Scoliosis	Paeds ENT	Vascular	Paeds Gen	CEPOD	Urology	Renal	Liver	Oral	Cardiac	Cardiac	Neuro	Neuro
FriPM	Trauma	Scoliosis	ENT	Vascular	Paeds Gen	CEPOD	Urology	Renal	Liver	Oral	Cardiac	Cardiac	Neuro	Neuro

- Optimal MSS: (example)

	Theatre 0	Theatre 1	Theatre 2	Theatre 3	Theatre 4	Theatre 5	Theatre 6	Theatre 7	Theatre 8	Theatre 9	Theatre 10	Theatre 11	Theatre 12	Theatre 14
MonAM	Trauma	Paeds Ortho	Paeds ENT	Renal	Paeds Gen	CEPOD	Urology	Renal	General	Thoracic	Cardiac	Cardiac	Neuro	Neuro
MonPM	Trauma	Paeds Ortho	ENT	Renal	Paeds Gen	CEPOD	Urology	Renal	General	Thoracic	Cardiac	Cardiac	Neuro	Neuro
TuesAM	Trauma	Scoliosis	ENT	Renal	Oral	CEPOD	Urology	Colorectal	General	Thoracic	Cardiac	Cardiac	Neuro	Neuro
TuesPM	Trauma	Scoliosis	ENT	Renal	Oral	CEPOD	Urology	Colorectal	General	Thoracic	Cardiac	Cardiac	Neuro	Neuro
WedAM	Trauma	Scoliosis	ENT	Vascular	Paeds Gen	CEPOD	Urology	Colorectal	General	Thoracic	Cardiac	Cardiac	Neuro	Neuro
WedPM	Trauma	Scoliosis	ENT	Vascular	Paeds Gen	CEPOD	Urology	Colorectal	General	Thoracic	Cardiac	Cardiac	Neuro	Neuro
ThurAM	Trauma	Vascular	Oral	Vascular	Paeds Gen	CEPOD	Urology	Colorectal	General	Thoracic	Cardiac	Cardiac	Neuro	Neuro
ThurPM	Trauma	Vascular	Oral	Vascular	Paeds Gen	CEPOD	Urology	Colorectal	General	Thoracic	Cardiac	Cardiac	Neuro	Neuro
FriAM	Trauma	Trauma	Ophthal	Vascular	Paeds Gen	CEPOD	Urology	Colorectal	Liver	Oral	Cardiac	Cardiac	Neuro	Neuro
FriPM	Trauma	Trauma	Ophthal	Vascular	Paeds Gen	CEPOD	Urology	Colorectal	Liver	Oral	Cardiac	Cardiac	Neuro	Neuro

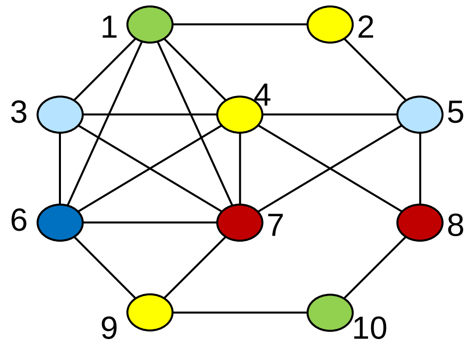
- Optimal schedule simulated 1000 times
  - Bed constraints were violated in 77% of these simulations

Multiple scenario results



Ongoing work

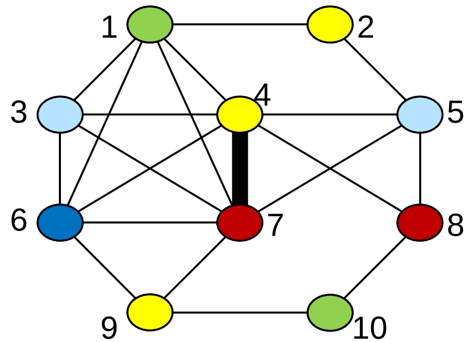
- Might adjust the level of conservatism of the robust solutions in terms of probabilistic bounds of constraint violations
    - *D. Bertsimas and M. Sim, The price of robustness,* Operations Research, *52 (2004), pp. 35-53.*
- Various *what-if* scenarios can now be investigated
- What about problems that are too big to solved exactly:
    - Perhaps use heuristics and metaheuristics
    - Make the robustness part of the objective function



Timeslot / Theatre	**1**	**2**	**3**	**4**	**5**
OT 1	2	10	8		
OT 2	4		7	6	3
OT 3	9	1			5

Ongoing work

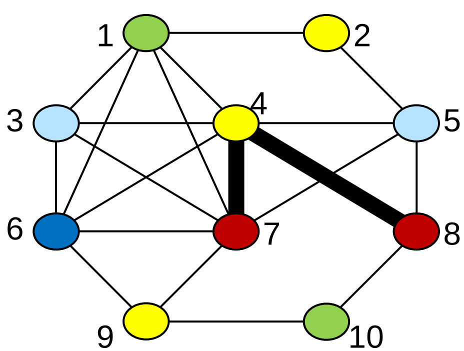
- Might adjust the level of conservatism of the robust solutions in terms of probabilistic bounds of constraint violations
    - *D. Bertsimas and M. Sim, The price of robustness,* Operations Research, *52 (2004), pp. 35-53.*
- Various *what-if* scenarios can now be investigated
- What about problems that are too big to solved exactly:
    - Perhaps use heuristics and metaheuristics
    - Make the robustness part of the objective function



Timeslot / Theatre	**1**	**2**	**3**	**4**	**5**
OT 1	2	10	8		
OT 2	**4**		**7**	6	3
OT 3	9	1			5

Ongoing work

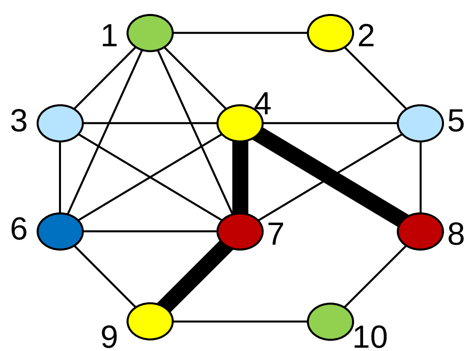
- Might adjust the level of conservatism of the robust solutions in terms of probabilistic bounds of constraint violations
  - *D. Bertsimas and M. Sim, The price of robustness,* Operations Research, *52 (2004), pp. 35-53.*
- Various *what-if* scenarios can now be investigated
- What about problems that are too big to solved exactly:
  - Perhaps use heuristics and metaheuristics
  - Make the robustness part of the objective function



Timeslot / Theatre	**1**	**2**	**3**	**4**	**5**
OT 1	2	10	**8**		
OT 2	**4**		**7**	6	3
OT 3	9	1			5

Ongoing work

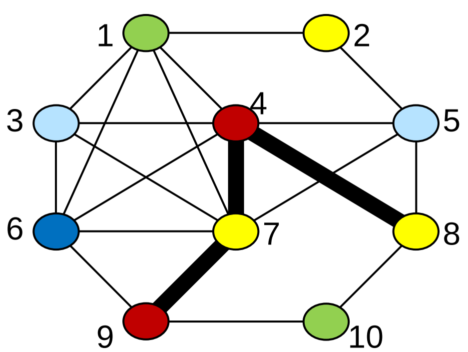
- Might adjust the level of conservatism of the robust solutions in terms of probabilistic bounds of constraint violations
    - *D. Bertsimas and M. Sim, The price of robustness,* Operations Research, *52 (2004), pp. 35-53.*
- Various *what-if* scenarios can now be investigated
- What about problems that are too big to solved exactly:
    - Perhaps use heuristics and metaheuristics
    - Make the robustness part of the objective function



Timeslot / Theatre	**1**	**2**	**3**	**4**	**5**
OT 1	2	10	**8**		
OT 2	**4**		**7**	6	3
OT 3	**9**	1			5

Ongoing work

- Might adjust the level of conservatism of the robust solutions in terms of probabilistic bounds of constraint violations
    - *D. Bertsimas and M. Sim, The price of robustness,* Operations Research, *52 (2004), pp. 35-53.*
- Various *what-if* scenarios can now be investigated
- What about problems that are too big to solved exactly:
    - Perhaps use heuristics and metaheuristics
    - Make the robustness part of the objective function

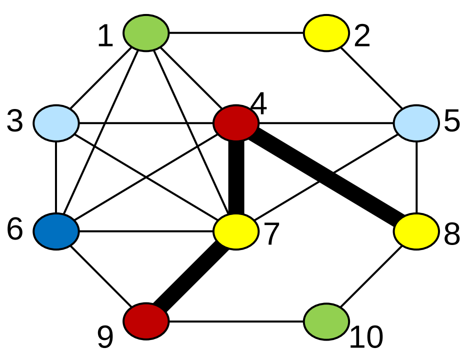


Timeslot / Theatre	**1**	**2**	**3**	**4**	**5**
OT 1	2	10	**4**		
OT 2	**7**			6	3
OT 3	**8**	1	**9**		5

Ongoing work

- Might adjust the level of conservatism of the robust solutions in terms of probabilistic bounds of constraint violations
    - *D. Bertsimas and M. Sim, The price of robustness, Operations Research, 52 (2004), pp. 35-53.*
- Various *what-if* scenarios can now be investigated
- What about problems that are too big to solved exactly
    - Perhaps use heuristics and metaheuristics
    - Make the robustness part of the objective function

Thank you.



Timeslot / Theatre	**1**	**2**	**3**	**4**	**5**
OT 1	2	10	**4**		
OT 2	**7**			6	3
OT 3	**8**	1	**9**		5