# Exact Bar Nesting with Industrial Symmetry Considerations

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## 1 Introduction

Companies working with steel, wood, or PVC bars are often interested in increasing the efficiency of their operations. To that end, they seek to reduce material waste by minimizing the number of stocks used for cutting the desired items. Extensive literature, typically termed "bin packing" or "stock cutting" [1, 2, 3], surrounds this problem. The well-known 1D cutting stock problem focuses on the manufacturing of bars (items) with straight cuts. In practice, however, straight cuts may not be appropriate. In this work, we study cases appearing in industries where one must cut bars with angled ends. In addition, we also consider (i) how bars can have different symmetry planes depending on their sections, (ii) how this affects the question of cutting effectively in each pattern, and (iii) what algorithms can be used for efficiently solving such problems.

## 2 Literature review

In the industry, the problem of cutting bars with angled ends is often referred to as the bar nesting problem. To cut bars efficiently, one can apply *rotation* along their horizontal axis (along their length) or *flipping*, a 180° rotation along their vertical axis (see figures 1 and 2). In some practical cases not yet considered in the literature, some bars can also have different sections, implying different symmetry planes. Ultimately, bar sections need to be acknowledged as they potentially yield limitations regarding (i) the rotation along the horizontal axis and (ii) the flip (see figure 3).

In previous work, Garraffa et al. [1] investigated a 1D cutting stock problem in which the wastage between successive items on a stock is calculated via an arbitrary function, but

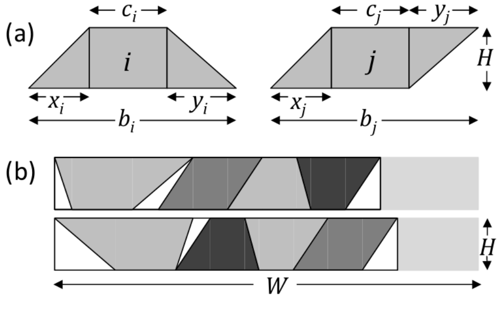


FIG. 1: (a) Bars variables ; and (b) two example of packing 4 bars with different 180° rotations and flips (figure from [2])

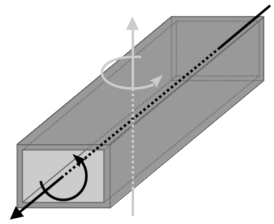

FIG. 2: Vizualisation of a bar rotation along its hozirontal axis (black) and vertical axis (light grey).

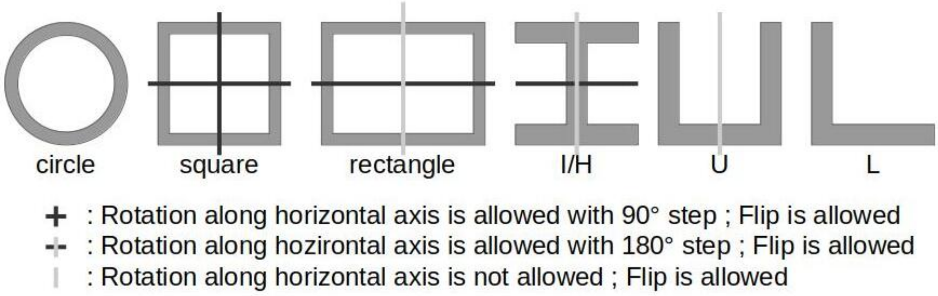


FIG. 3: Sections

where bars can only be rotated along their horizontal axis. This formulation is actually a generalization of the $\mathcal{NP}$-hard traveling salesman problem. In contrast, Lewis et al. [2] consider rotation along the vertical axis but do not consider symmetry planes. They model the problem as a trapezoid packing problem in which wastage between successive items can be derived using simple 2D geometry. This allows the creation of a polynomial-time algorithm to answer the question of how items can be optimally arranged onto a single stock.

## 3 Technical work

We extend the work of [2] to account for symmetry issues arising in the industry. We use the term *matching issue* to describe when two bars $i$ and $j$ cannot have their angled ends $y_i$ and $x_j$ nested together (as is the case in [2]). *Matching issues* result from symmetry issues leading to a restricted rotation along the horizontal axis of the bars. We show that, in that context, the inter-item wastage between the two ends can be simplified to $|y_i + x_j|$. We then show that the resultant problem can be modeled as a minimum-cost perfect matching problem, leading to a polynomial-time method operating with an $O(n^3)$ time complexity. In cases where the rotation along the vertical axis is disabled, we also show that the problem can be solved by adapting the work of [2] to use directed graphs instead of undirected graphs. Finally, we improve on the work of [2] by decreasing the time complexity of the method from $O(n^3)$ to $O(n^2)$.

## 4 Conclusions and Perspectives

This preliminary paper focuses on improving and adapting the work of [2] to allow the arrangement of items in the context of the industrial bar nesting problem. In the future, we intend to test the performances of these methods in the context of the global optimization problem of producing patterns well suited for the industry.

## References

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